

**Grade 11**  
**Applied Mathematics 30S**

**Midterm Examination**

**January 29, 2014**

Name: KEY.

Final Mark: /55 = \_\_\_\_\_%



**Multiple Choice (15 marks)**

Identify the choice that best completes the statement or answers the question.

D 1. Bob made the following conjecture:

The difference between two numbers always lies between the two numbers.

Is the following equation a counterexample to this conjecture? Explain.

$$6 - (-2) = 8$$

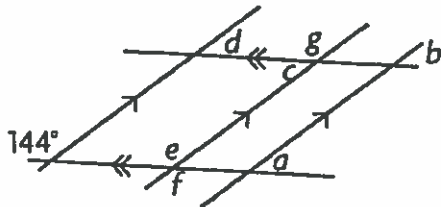
- a. Yes, it is a counterexample, because 8 lies between  $-2$  and  $6$ .
- b. No, it is not a counterexample, because 8 lies between  $-2$  and  $6$ .
- c. No, it is not a counterexample, because 8 is greater than  $-2$  and  $6$ .
- d. Yes, it is a counterexample, because 8 is greater than  $-2$  and  $6$ .

A 2. What type of error, if any, occurs in the following deduction?

All people drive cars to work.  
Gavin drives to work.  
Therefore, Gavin drives a car.

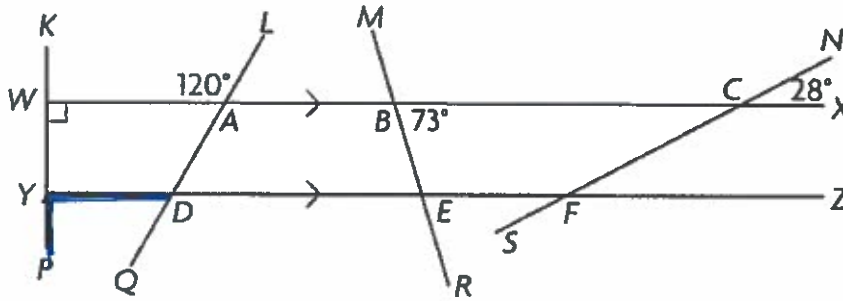
- a. a false assumption or generalization
- b. an error in reasoning
- c. an error in calculation
- d. There is no error in the deduction.

D 3. Which statement about the angles in this diagram is false?



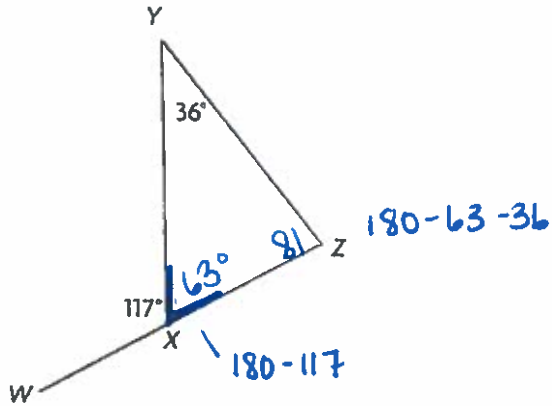
- a.  $\angle e = \angle f$
- b.  $\angle a = \angle b$
- c.  $\angle d = \angle c$
- d.  $\angle f = \angle a$

A 4. Which angle property proves  $\angle PYD = 90^\circ$ ?



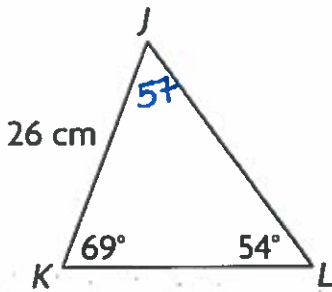
- a. corresponding angles
- b. alternate interior angles
- c. alternate exterior angles
- d. supplementary angles

C 5. Which are the correct measures for  $\angle YXZ$  and  $\angle XZY$ ?



- a.  $\angle YXZ = 63^\circ, \angle XZY = 91^\circ$
- ~~b.~~  $\angle YXZ = 53^\circ, \angle XZY = 91^\circ$
- c.  $\angle YXZ = 63^\circ, \angle XZY = 81^\circ$
- ~~d.~~  $\angle YXZ = 53^\circ, \angle XZY = 81^\circ$

- D 6. Determine the length of  $k$  to the nearest centimetre.



~~$\frac{26}{\sin 54} = \frac{x}{\sin 69}$~~

$$\frac{26}{\sin 54} = \frac{x}{\sin 69}$$

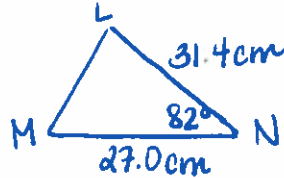
$$x = \frac{26 \sin 69}{\sin 54}$$

$$= 30$$

- a. 29 cm  
b. 28 cm  
c. 27 cm  
d. 30 cm

- B 7. In  $\triangle LMN$ ,  $l = 27.0$  cm,  $m = 31.4$  cm, and  $\angle N = 82^\circ$ . Determine the measure of  $n$  to the nearest tenth of a centimetre.

- a. 39.0 cm  
b. 38.5 cm  
c. 39.5 cm  
d. 38.0 cm



$$n^2 = 31.4^2 + 27.0^2 - 2(31.4)(27.0)\cos 82$$

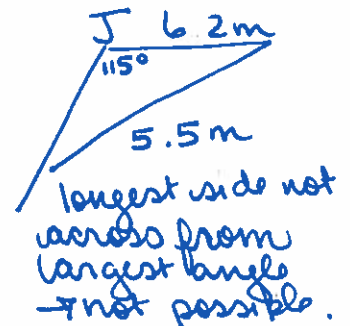
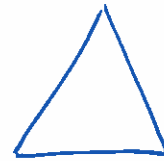
$$= 1714.96 - 235.98$$

$$n = \sqrt{1478.98}$$

$$= 38.5 \text{ cm}$$

- B 8. In  $\triangle JKL$ ,  $\angle J = 115^\circ$ ,  $k = 6.2$  m, and  $j = 5.5$  m. Which statement is true for this set of measurements?

- a. This is not a SSA situation.  
b. This is a SSA situation; no triangle is possible.  
c. This is a SSA situation; only one triangle is possible.  
d. This is a SSA situation; two triangles are possible.



- A 9. Determine the mean of the following test scores.

History Test 1 Scores (out of 100)

90	84	77	66
89	84	77	65
86	82	75	65
86	81	72	61
84	79	70	56

$$\frac{\Sigma}{n} = \frac{1529}{20} = 76.45$$

- a. 76.45  
b. 79.2  
c. 78.5  
d. 74.25

C

10. A company measured the lifespan of a random sample of 30 light bulbs. Times are in hours.

985	✓1001	✓1024	1087	952
910	938	931	1074	1081
1078	1080	982	1108	✓1022
937	922	✓1017	1093	1115
880	✓1048	917	1086	935
936	986	✓1038	954	966

What value goes in the fourth row of this frequency table?

Lifespan (hours)	Frequency
850-899	1
900-949	8
950-999	6
1000-1049	<b>5</b>
1050-1099	7
1100-1149	2

or:  $30 - (1 + 8 + 6 + 7 + 2)$   
 $= 30 - 24$   
 $= 6$

- a. 5
- b. 3
- c. 6
- d. 4

C

11. At the end of a bowling tournament, three friends analyzed their scores.

Lada's mean bowling score is 125 with a standard deviation of 27.

Quinn's mean bowling score is 182 with a standard deviation of 28.

Kamal's mean bowling score is 170 with a standard deviation of 20.

Who is the more consistent bowler?

- a. Impossible to tell.
- b. Quinn
- c. Kamal
- d. Lada

consistency = lower  $\sigma$

D

12. A set of data is normally distributed. What percent of the data is within two standard deviation of the mean?

- a. about 68%
- b. 100%
- c. about 50%
- d. about 95%

D

13. Determine the z-score for the given value.

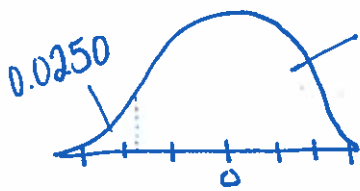
$\mu = 184, \sigma = 8.6, x = 174$

- a. -0.86
- b. 0.86
- c. 1.16
- d. -1.16

$$z = \frac{x - \mu}{\sigma} = \frac{174 - 184}{8.6} = -1.16$$

B 14. Determine the percent of data to the right of the z-score:  $z = -1.96$ .

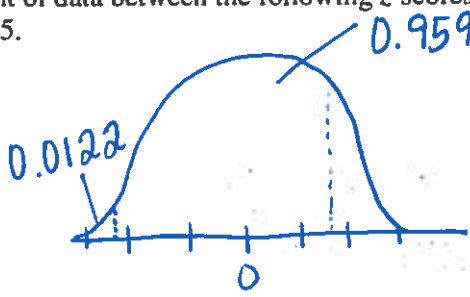
- a. 98.50%
- b. 97.50%
- c. 1.50%
- d. 2.50%



$$0.9750 = 97.5\%$$

B 15. Determine the percent of data between the following z-scores:  $z = -2.25$  and  $z = 1.75$ .

- a. 95.99%
- b. 94.77%
- c. 93.55%
- d. 97.23%



$$\begin{array}{r} 0.9599 \\ - 0.0122 \\ \hline 0.9477 \\ = 94.77\% \end{array}$$

**Short Answer (26 marks)**

16. Tyler made the following conjecture:

A polygon with more than two right angles must be a rectangle.

Do you agree or disagree? Briefly justify your decision with a counterexample if possible. (1)

*Disagree. Ex:*



*This is a polygon with more than two right angle that is not a rectangle.*

17. What error occurs in the following deduction? Circle it (1), then write a correct version. (1)

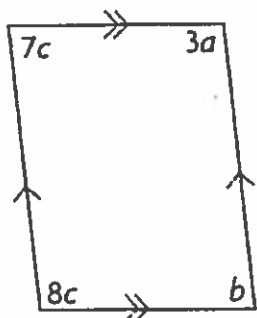
$$\begin{array}{l} 3 = 3 \\ 7(3) = 7(2+1) \\ 7(3) + 6 = 7(2+1) + 6 \\ 21 + 6 = 14 + 7 \\ 27 = 21 \end{array}$$

$$\begin{array}{l} 3 = 3 \\ 7(3) = 7(2+1) \\ 7(3) + 6 = 7(2+1) + 6 \\ 21 + 6 = 14 + 7 + 6 \\ 27 = 27 \end{array}$$

18. Determine the unknown term in this pattern. (1)

12, 7, 14, 9, 16, 11, 18      -5, +7

19. Determine the values of  $a$ ,  $b$ , and  $c$ . (3)



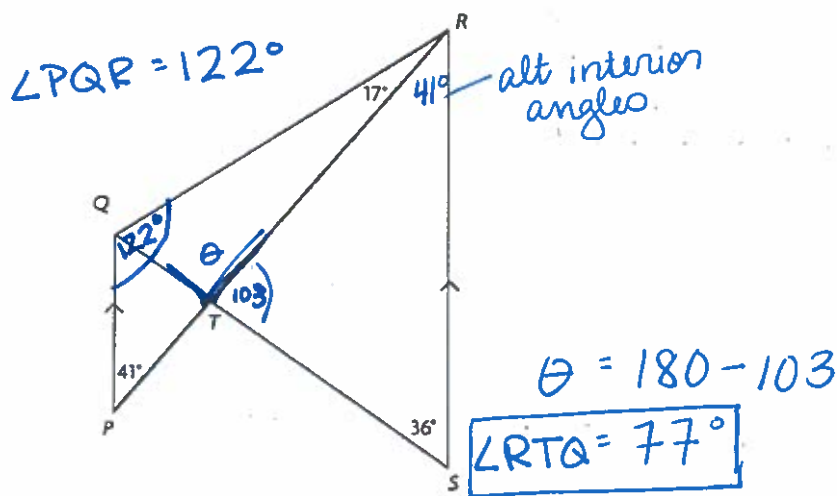
$$\begin{array}{l} 7c + 8c = 180 \\ 15c = 180 \\ \frac{15c}{15} = \frac{180}{15} \\ c = 12^\circ \end{array}$$

$$\begin{array}{l} 3a = 8c \\ a = \frac{8(12)}{3} \\ a = 32^\circ \end{array}$$

$$\begin{array}{l} 7c = b \\ 7(12) = b \\ b = 84^\circ \end{array}$$

$$\begin{array}{l} a = 32^\circ \\ b = 84^\circ \\ c = 12^\circ \end{array}$$

20. Determine the measure of  $\angle RTQ$ . (2)



21. Each interior angle of a regular convex polygon measures  $156^\circ$ . How many sides does the polygon have? (1)

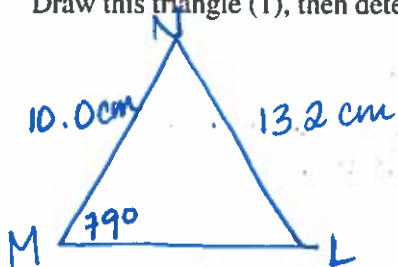
$$\frac{(n-2)180}{n} = \frac{n(156)}{n}$$

$(n-2)180 = \text{sum of interior angles}$   
 $\text{sum of interior angles} = \text{each angle} \times n$   
 $n(\text{each angle}) = \sum \text{int } \angle\text{'s}$

$$\begin{aligned} (n-2)180 &= n(156) \\ 180n - 360 &= 156n \\ 180n - 156n &= 360 \\ n &= 15 \end{aligned}$$

$\boxed{n = 15}$     $\boxed{15 \text{ sides}}$

22. In  $\triangle LMN$ ,  $l = 10.0$  cm,  $m = 13.2$  cm, and  $\angle M = 79^\circ$ . Draw this triangle (1), then determine the measure of  $\angle L$  to the nearest degree. (2)

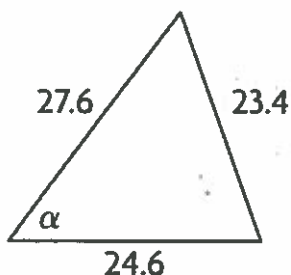


$$\frac{13.2}{\sin 79} = \frac{10.0}{\sin L}$$

$$\frac{10.0 \sin 79}{13.2} = \sin L$$

$\boxed{\angle L = 48^\circ}$

23. Determine the measure of  $\alpha$  to the nearest degree. (2)



$$\cos \alpha = \frac{23.4^2 - 27.6^2 - 24.6^2}{-2(27.6)(24.6)}$$

$$\alpha = \cos^{-1}(0.60339)$$

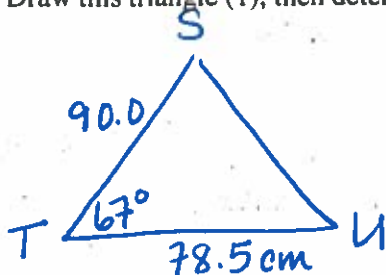
$$= 52.9^\circ$$

$\boxed{= 53^\circ}$

8/



24. In  $\Delta STU$ ,  $s = 78.5$  cm,  $u = 90.0$  cm, and  $\angle T = 67^\circ$ .  
 Draw this triangle (1), then determine the measure of  $t$  to the nearest tenth of a centimetre. (2)



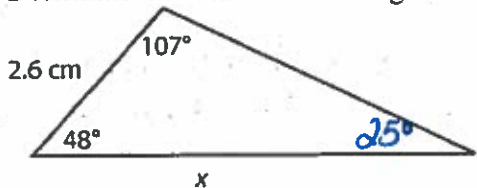
$$t^2 = 90.0^2 + 78.5^2 - 2(90.0)(78.5)\cos 67$$

$$t^2 = 14262.25 - 5521.03$$

$$t = \sqrt{8741.22}$$

$$t = 93.5 \text{ cm}$$

25. Determine the unknown side length to the nearest tenth of a centimetre. (2)



$$\frac{2.6}{\sin 25} = \frac{x}{\sin 107}$$

$$x = \frac{2.6 \sin 107}{\sin 25}$$

$$x = 5.9 \text{ cm}$$

26. Joel researched the average daily temperature in his town.  
 Average Daily Temperature in Lloydminster, SK

Month	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
average daily temperature (°C)	-10.0	-17.5	-5.0	3.7	10.7	14.3	20.1	14.0	9.8	4.8	-5.8	-14.8
	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

Determine the median of the data. (1)

-17.5, -14.8, -10.0, -5.8, -5.0, 3.7, 4.8, 9.8, 10.7, 14.0, 14.3, 20.1

$$\frac{3.7 + 4.8}{2} = 4.25 = \text{median}$$

27. Environment Canada compiled data on the number of lightning strikes per square kilometre in Saskatchewan and Manitoba towns from 1999 to 2008.

highest: 2.03

2.03	1.31	0.25	1.03	1.20	0.17
0.99	1.01	0.24	0.94	0.92	0.09
0.86	0.71	0.05	0.81	0.63	0.01
0.80	0.58	0.00	0.72	0.49	0.52
0.43	0.46	0.40			

If the interval width is 0.25 and starts at 0.00, how many intervals are there? (1)

0.00 - 0.25      1.00 - 1.25  
 0.25 - 0.50      1.25 - 1.50  
 0.50 - 0.75      1.50 - 1.75  
 0.75 - 1.00      1.75 - 2.00  
 2.00 - 2.25

there are 9 intervals

16

28. The ages of members in a hiking club are normally distributed, with a mean of 32 and a standard deviation of 6 years. What percent of the members are younger than 20? (2)

$$z = \frac{x - \mu}{\sigma} \quad \mu = 32 \quad \sigma = 6 \quad z = ? \quad x = 20$$

$$z = \frac{20 - 32}{6} = -2$$

2.5% of members are younger than 20.

or: 2.28% if you use z-score table

29. A teacher is analyzing the class results for a computer science test. The marks are normally distributed with a mean ( $\mu$ ) of 79.5 and a standard deviation ( $\sigma$ ) of 3.5. Determine Carmine's mark if he scored  $\mu - \sigma$ . (1)

$$\begin{aligned} \mu - \sigma &= 79.5 - 3.5 \\ &= 76.0 \end{aligned}$$

Carmine's mark was 76.0.

30. Determine the percent of data to the left of the z-score:  $z = 0.71$ . (1)

$$0.7611 = 76.11\%$$

**Problem (14 marks)**

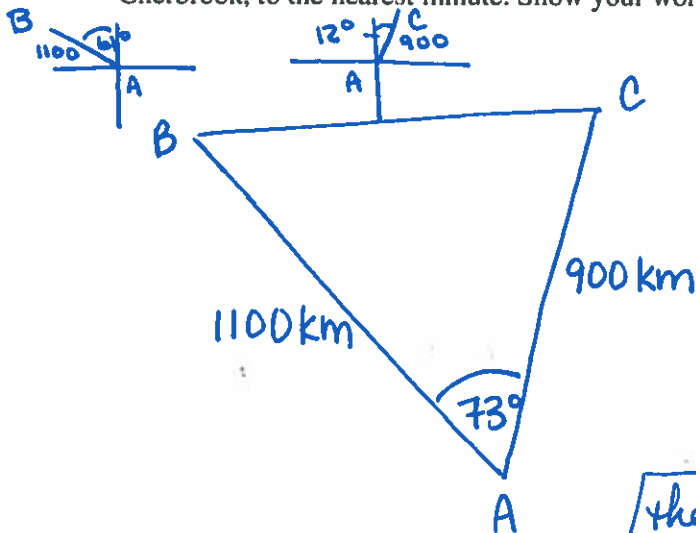
31. Paul made the following conjecture:

The square of a number is always greater than the original number.

John disagreed with Paul's conjecture, however, because  $0.4^2 = 0.16$ , and 0.16 is less than 0.4. How could Paul's conjecture be improved? Explain the changes you would make. (2)

Ex / The square of a number greater than one or less than zero is always greater than the original number.

32. A pilot leaves Bayridge Airport and flies to Cherbrook at a constant speed of 400 km/h. She knows that the distance from Allenburg to Bayridge is 1100 km N61°W and the distance from Allenburg to Cherbrook is 900 km N12°E. Determine the **duration** of the flight from Bayridge to Cherbrook, to the nearest minute. Show your work. (4)



$$a^2 = 900^2 + 1100^2 - 2(900)(1100)\cos 73$$

$$a^2 = 2020000 - 578895.97$$

$$a = \sqrt{1441104.025}$$

$$a = 1200.46 \text{ km}$$

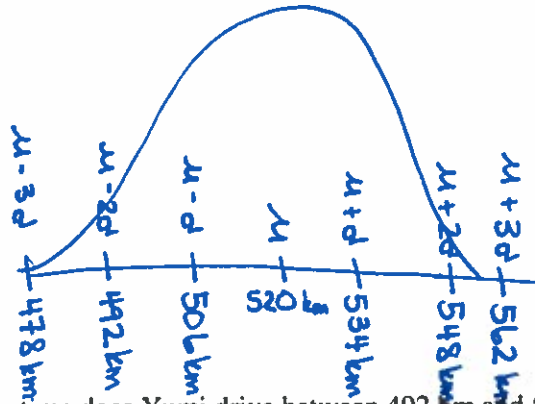
$$\frac{1200.46 \text{ km}}{400 \text{ km/h}} = 3.0011 \text{ h}$$

$$= 3 \text{ hours.}$$

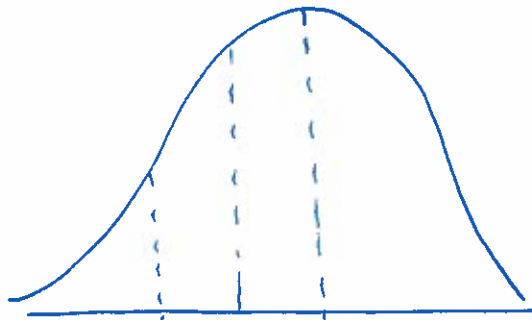
the flight duration is 3 hours

33. Yumi always waits until her gas tank is nearly empty before refuelling. She keeps track of the distance she drives on each tank of gas. The distance varies depending on the weather and the amount she drives on the highway. The distance has a mean of 520 km and a standard deviation of 14 km.

a) Sketch a normal curve to show the distribution of the driving distances for a tank of gas. Mark the kilometres driven for values that are 1, 2, and 3 standard deviations from the mean. (2)



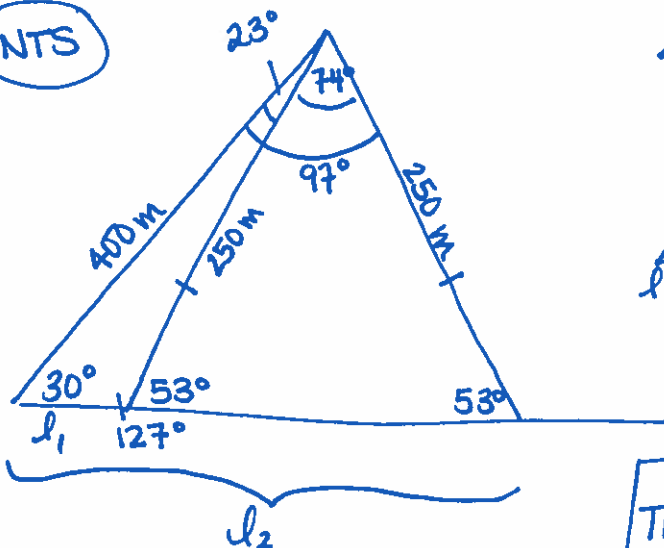
b) What percent of the time does Yumi drive between 492 km and 520 km on a tank of gas? (2)



$$\frac{95}{2} = 47.5\% \text{ of the time}$$

34. A landowner claims that her property is triangular, with one side 400 m long and another side 250 m long. The opposite angle to the 250 m side measures  $30^\circ$ . Determine two possible lengths of the third side, to the nearest metre. Draw the two triangles, and show all your work. (4)

NTS



$$\frac{250}{\sin 30} = \frac{400}{\sin \theta}$$

$$\theta = \sin^{-1}\left(\frac{400 \sin 30}{250}\right) = 53^\circ$$

$$\frac{250}{\sin 30} = \frac{x}{\sin 23}$$

$$x = \frac{250 \sin 23}{\sin 30}$$

$$x = 195 \text{ m } (l_1)$$

or

$$180 - 53 = 127^\circ$$

$$\frac{250}{\sin 30} = \frac{x}{\sin 97}$$

$$x = \frac{250 \sin 97}{\sin 30}$$

$$x = 496 \text{ m } (l_2)$$

The third side is either 195 m or 496 m.

